3D-FDTD SUBGIDDING TECHNIQUE APPLIED TO RADIATING STRUCTURES

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***Abstract* — A 3D-FDTD subgridding technique based on a finite element formulation is presented. This subgridding technique is applied to compute the input impedance of a mobile phone antenna. The obtained results are in good agreement with those obtained using a uniform mesh, while the CPU time and the used computer memory are largely less important.**

I. INTRODUCTION

The popular Finite Difference Time Domain (FDTD) method uses a uniform orthogonal structured mesh to describe the geometry of studied structures. This structured mesh, upon which the success of this technique is based, has a drawback when curved or fine objects have to be studied. In such cases, the mesh has to be fine in the entire domain to avoid errors due to "staircase approximation". However in most of the studied problems geometrically-difficult sub-domains are not spread throughout the computational domain. Then a uniform fine mesh leads to a waste of computer memory and CPU time. This is the reason for which subgridding techniques are proposed as an efficient tool to improve the FDTD [1]. They consist in using a fine mesh only in the geometrically-critical areas and a coarse mesh elsewhere. The validity of a subgridding scheme relies on the proper processing of the fine mesh/coarse mesh interfaces. For instance, stability is a key issue.

II. SUBGRIDDING METHOD

In our subgridding method, FDTD is considered as a special case of Finite Element Time Domain (FETD) method, using structured mesh and mass lumping [2]. This method was initially developed in the 2D case [3] and then extended to the 3D case [4].

A. Basic Equations

Basically, the problem is to compute the evolution of the electromagnetic field in a given domain Ω splitted into two computational domains Ω_c and Ω_f (with $\Omega_c \cup \Omega_f = \Omega$) meshed independently, even on their common interface (figure 1).

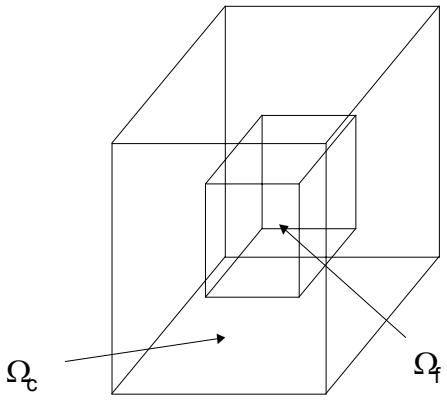

Fig. 1. Geometry of the problem.

In each domain, Maxwell curl equations must be verified:

$$\begin{aligned} &\varepsilon\partial_t E_f - rotH_f = 0 \\ &\mu_0\partial_t H_f + rotE_f = 0 \\ &\varepsilon\partial_t E_c - rotH_c = 0 \\ &\mu_0\partial_t H_c + rotE_c = 0 \end{aligned} \quad (1)$$

The matching condition between both domains is the continutiy of tangential electric field and normal magnetic field over the meshes interface. For notation convenience, the normal magnetic field continuity is ensured through "currents" defined by $J = n \wedge H$, where n is a vector normal to the meshes interface:

$$n \wedge E_f = n \wedge E_c \quad (2)$$

$$J_c + J_f = 0 \quad (3)$$

Equations (1), (2) and (3) are then multiplied by a "test function" and integrated over the computational domain, in order to obtain the weak formulation of the problem. It is worth to mention, from a mathematical point of view, that the electric currents J_c and J_f can be considered as the Lagrange multipliers associated to the problem [5].

B. Spatial discretization

Fields, currents, and test functions are then developed on a base of mixed (edge/face) elements. This Galerkin process leads to a set of matrices equations:

$$\begin{aligned} &R_k e_k + \partial_t b_k = 0 \quad k \in \{c, f\} \\ &{}^t R_k \mu_k^{-1} b_k - \partial_t \varepsilon_k e_k + T_k J_k = 0 \quad k \in \{c, f\} \\ &M_c e_{tc} - M_c e_{tc} = 0 \\ &N_{fc} e_{tc} - M_f e_{tf} = 0 \\ &M_c J_c + {}^t N_{fc} J_f = 0 \end{aligned} \quad (4)$$

In these equations, R_c and R_f are discrete curl operators, ε_c and ε_f describe the interaction between edges of the coarse mesh and those of the fine mesh respectively, μ_c and μ_f describe the interaction between faces of the coarse mesh and those of the fine mesh respectiveley, M_c and M_f describe the interaction between edge elements located on the meshes interface on the coarse mesh and on the fine mesh respectively, N_{fc} relates the edge elements of the coarse mesh to the fine one on the interface (it is a kind of "projection" matrix), T_c and T_f are space matrices whose subblocks are respectively M_c and M_f. One of the key points of the method can be seen in the equation $M_c e_{tc} - M_c e_{tc} = 0$, which expresses the fact that one must choose a reference for the representation of the electric field on the interface. In this work, the coarse representation was chosen.

C. Mass lumping

In order to diagonalize all matrices, a mass lumping process is performed. In the computation of matrices elements, this process consists of replacing continuous integration by discrete summation over the vertices of the elements. This is equivalent to modify the shape of the edge elements, as shown in figure 2. The lumping process is also performed for matrices involving 2D elements at the meshes interface.

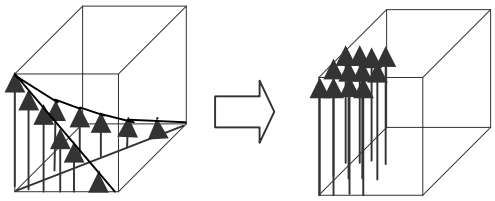

Fig. 2 Effect of the mass lumping process on the edge elements.

D. Time discretization

To compute the time evolution of the fields, a leap-frog time scheme is adopted [6]. Note that for stability reasons, the time step in the fine meh is not necessarily the same as that in the coarse one: we can

talk of space-time subgridding. Our formulation shows that the classical Yee scheme is obtained inside both domains, while a modified Yee scheme is obtained at the meshes interfaces. Figure 3 illustrates the interdependence between fields by representing the first steps of the time scheme, in a case of a time step reduction factor of 3. In this figure, capital letters represent fields in the coarse mesh while small letters represent fields in the fine mesh. A field placed at the starting point of an arrow is used to update the field placed at the arrowhead. Large vertical arrows show the tangentiel electric field projection from "fine side" to "coarse side" at the meshes interface. The scheduling of fields updates and projections is indicated by the numbers associated to each arrow.

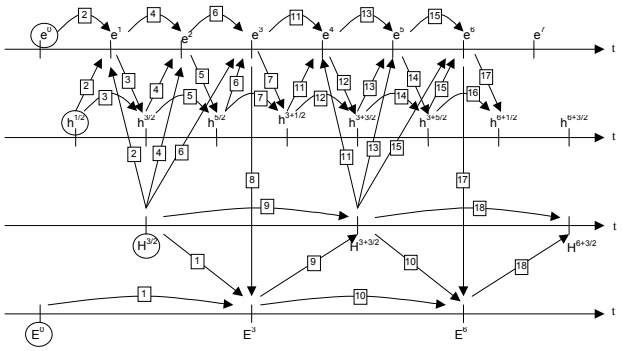

Fig 3. Subgridding time scheme, showing the interdependence between fields.

III. TEST CASE

The subgridding scheme proposed above is used to compute the input impedance at 900 MHz of a GSM cellular phone antenna. The structure is modeled as a quarter wavelength dipole placed upon a metallic box (figure 4). The antenna is excited by a gap of length d=0.5 cm which is placed juste above the box. The size of the FDTD cell in the direction of the antenna cannot be larger than the gap length, which is very small compared to the wavelength (d=λ/66 at the frequency of operation). Using a uniform mesh, this leads to a globally over-precised mesh.

The structure is firstly meshed using a uniform 5 mm x 5 mm x 5 mm mesh, the phone being at the center of a 2λ x 2λ x 2.5λ box covered by Mur 1st order type absorbing boundary conditions (figure 5). The input impedance Z_1 is calculated and found to be 44,05 – j 21,65 Ω. The structure is then studied using the subgridding scheme. The phone is meshed using the same mesh dimensions as above, while the peripherical region is meshed three times coarser. The input impedance is found to be Z_2=43.56 – j 21.80 Ω. We can see that Z_1 and Z_2 are nearly similar. A difference of 1.1 % for the real part is calculated. The results are also in good agreement with those obtained by [7] using a finite element analysis in the frequency domain. The simulation in the sub-gridded case takes 7 times less memory and 14 times less CPU time than the uniform one. Then, we performed the calculation of the input impedance over a broad frequency band. As can be seen from figure 6 results for the real part of the input impedance using a uniform mesh and those using our proposed subgridding technique are nearly coincident.

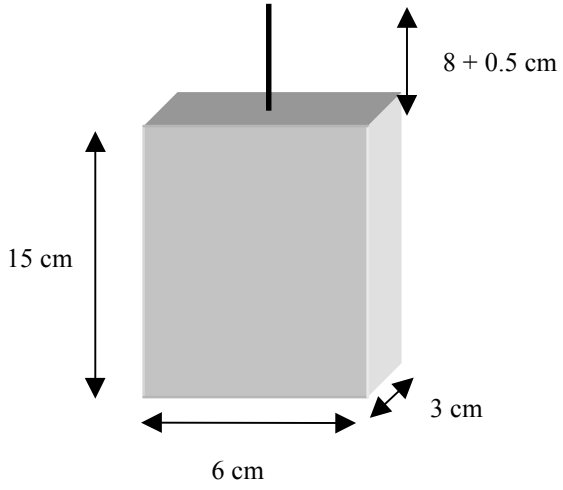

Fig 4. Mobile phone model.

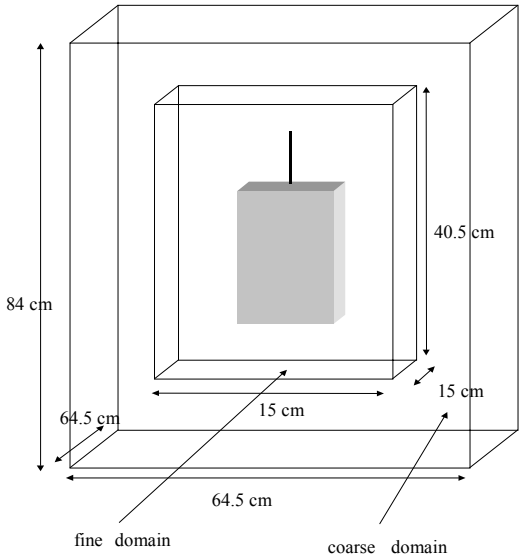

Fig. 5 Position of the interfaces for the studied mobile phone antenna.

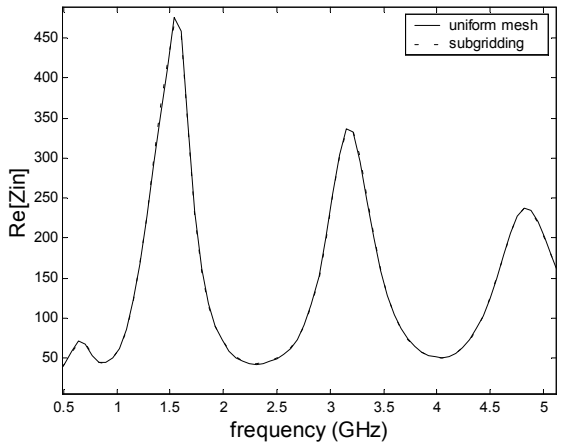

Fig. 6 Variation of the real part of the input impedance of a mobile phone antenna as a function of frequency.

IV. CONCLUSION

The proposed FEM based FDTD subgridding scheme is shown to be an efficient technique that allows the electromagnetic characterization of problems having different geometrically difficult subdomains. It leads to the use of largely less computer memory and CPU time than the case when uniform mesh is used for the global domain discretisation. The validity of our proposed subgridding scheme is demonstrated through its application on the test case of a mobile phone antenna and the comparison of the results using our technique to those obtained using uniform mesh. The proposed formulation could be applied in the case where one of the two meshes is unstructured (tetrahedral) while the other is structured (parallelepipedical). The technique can then be seen as a finite element/ finite differences hybridation scheme, which would allow even more flexibility in the geometry of studied strucutres.

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